

# The Energy of the Gamma Metric in the Møller Prescription

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## ABSTRACT

We obtain the energy distribution of the gamma metric using the energy-momentum complex of Møller. The result is the same as obtained by Virbhadra in the Weinberg prescription.

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# 1 INTRODUCTION

Energy-momentum is regarded as the most fundamental conserved quantity in physics, and associated with a symmetry of space-time geometry. According to Noether's theorem and translations invariance, one could define a conserved energy-momentum  $T^{\mu\nu}$  as a consequence of its satisfying the differential conservation law  $\partial_\nu T^{\mu\nu} = 0$ . However, in a curve space-time where the gravitational field is presented, the differential conservation law becomes

$$\nabla_\nu T^{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\nu} (\sqrt{-g} T^{\mu\nu}) - \frac{1}{2} g^{\nu\rho} \frac{\partial g^{\nu\rho}}{\partial x^\lambda} T^{\mu\lambda} = 0, \quad (1)$$

and generally does not lead to any conserved quantity. Early energy-momentum investigations attempted to determine the conserved energy-momentum for the gravitational field and the matter located in it, and led to energy-momentum complex

$$\Theta^{\mu\nu} = \sqrt{-g} (T^{\mu\nu} + t^{\mu\nu}), \quad (2)$$

which satisfies the differential conservation equation  $\partial_\nu \Theta^{\mu\nu} = 0$ . Here,  $T^{\mu\nu}$  is the energy-momentum tensor of matter and  $t^{\mu\nu}$  is regarded as the contribution of energy-momentum from the gravitational field. There are various energy-momentum complexes, including those of Einstein [1], Tolman [2], Papapetrou [3], Bergmann [4], Landau and Lifshitz [5], Møller [6], and Weinberg [7]. On the other way, a different idea, quasilocal (i.e., associated with a closed 2-surface) was proposed. The Hamiltonian for a finite region,

$$H(N) = \int_\Sigma N^\mu \mathcal{H}_\mu + \oint_{S=\partial\Sigma} \mathcal{B}(N), \quad (3)$$

generates the space-time displacement of a finite spacelike hypersurface  $\Sigma$  along a vector field  $N^\mu$ . Noether's theorem guarantee that  $\mathcal{H}_\mu$  is proportional to the field equation. Consequently, the value depends only on the boundary term  $\mathcal{B}$ , which gives the quasilocal energy-momentum. Moreover, there are also a large number of definitions of quasilocal mass [8, 9]. In their recent article, Chang *et al.* [9] showed that every energy-momentum complex can be associated with a particular Hamiltonian boundary term. So the energy-momentum complexes may also be considered as quasilocal.

Though Penrose [10] points out that a quasilocal mass is conceptually important. However, Bergqvist [11] studied several different definitions of

quasilocal masses for the Reissner- Nordström and Kerr space-times and came to the conclusion that not even two of these definitions gave the same results. On the contrary, several energy-momentum complexes have been showing a high degree of consistency in giving the same energy distribution for a given space-time. Recently, Virbhadra and his collaborators [12, 13, 14, 15, 16] have investigated that for a given space-time (like as the Kerr- Newman, the Vaidya, the Einstein-Rosen, the Bonnor-Vaidya and all Kerr-Schild class space-time) different energy-momentum complexes (the Einstein, the Landau-Lifshitz, the Papapetrou, the Tolman, The Weinberg, etc.) give the same energy distribution. Moreover some interesting results [12, 17, 18, 19, 20] led to the conclusion that in a given space-time (the Reissner-Nordström, the Kerr-Newman, the Garfinkle-Horowitz-Strominger, the de Sitter-Schwarzschild, and the charged regular metric, etc.) the energy distribution according to the energy-momentum complex of Møller is different from of Einstein. But in some specific case [6, 17] (the Schwarzschild, the Janis-Newman-Winicour metric, etc.) there are the same. Recently, the energy distribution in the Weinberg prescription obtained by Virbhadra [21] using the gamma metric, is given as

$$E = m \gamma. \quad (4)$$

So, in this letter, we evaluate the energy distribution of the gamma metric by using Møller energy-momentum complex, and compare with the result obtained by Virbhadra with Weinberg energy-momentum complex.

## 2 ENERGY IN THE MØLLER PRESCRIPTION

First, the well-known gamma metric [21, 22], a static and asymptotically flat exact solution of Einstein vacuum equations, is given as

$$ds^2 = \left(1 - \frac{2m}{r}\right)^\gamma dt^2 - \left(1 - \frac{2m}{r}\right)^{-\gamma} \left[ \left(\frac{\Delta}{\Sigma}\right)^{\gamma^2-1} dr^2 + \frac{\Delta^{\gamma^2}}{\Sigma^{\gamma^2-1}} d\theta^2 + \Delta \sin^2 \theta d\phi^2 \right], \quad (5)$$

where

$$\begin{aligned} \Delta &= r^2 - 2mr, \\ \Sigma &= r^2 - 2mr + m^2 \sin^2 \theta. \end{aligned} \quad (6)$$

For  $|\gamma| = 1$  the metric is spherically symmetric and for  $|\gamma| \neq 1$ , it is axially symmetric. In the situation  $|\gamma| = 1$ , the gamma metric reduces to the Schwarzschild space-time. However, in another situation  $|\gamma| \neq 1$ , the gamma metric gives the Schwarzschild space-time with negative mass, as putting  $m = -M$  ( $M > 0$ ) and carrying out a coordinate transformation  $r \rightarrow R = r + 2M$  one gets the Schwarzschild space-time with positive mass.

Next, let us consider the Møller energy-momentum complex which is given by [6]

$$\Theta_{\nu}{}^{\mu} = \frac{1}{8\pi} \frac{\partial \chi_{\nu}{}^{\mu\sigma}}{\partial x^{\sigma}}, \quad (7)$$

where the Møller superpotential,

$$\chi_{\nu}{}^{\mu\sigma} = \sqrt{-g} \left( \frac{\partial g_{\nu\alpha}}{\partial x^{\beta}} - \frac{\partial g_{\nu\beta}}{\partial x^{\alpha}} \right) g^{\mu\beta} g^{\sigma\alpha}, \quad (8)$$

are quantities antisymmetric in the indices  $\mu, \sigma$ . According to the definition of the Møller energy-momentum complex, the energy component is given as

$$\begin{aligned} E &= \int \Theta_0{}^0 dx^1 dx^2 dx^3 \\ &= \frac{1}{8\pi} \int \frac{\partial \chi_0{}^{0k}}{\partial x^k} dx^1 dx^2 dx^3, \end{aligned} \quad (9)$$

where the Latin index takes values from 1 to 3. However, in the case, the only nonvanishing component of Møller's superpotential is

$$\chi_0{}^{01} = 2m\gamma \sin\theta. \quad (10)$$

Applying the Gauss theorem to (9) and using (10), we evaluate the integral over the surface of a sphere with radius  $r$ , and find the energy distribution is

$$E = m\gamma. \quad (11)$$

It is the same result as obtained by Virbhadra in the Weinberg prescription.

### 3 DISCUSSION

It is well-known that the subject of the energy-momentum localization is associated with much debate. In contradiction with Misner *et al.*[23], Cooperstock and Sarracino [24] gave their viewpoint that if the energy localization is meaningful for spherical system it is, also, meaningful for all systems. Also,

Cooperstock [25] gave his opinion that the energy and momentum are confined to the regions of non-vanishing energy-momentum tensor of the matter and all non-gravitational fields. Bondi [26] sustained that a nonlocalizable form of energy is not admissible in relativity so its location can in principle be found.

We calculate the energy distribution of the gamma metric using the energy-momentum complex of Møller. The energy depends on the mass  $m$ . Thus, we get the same result as Virbhadra [21] obtained using the energy-momentum complex of Weinberg. This result sustains the opinion that different energy-momentum complexes could give the same expression for the energy distribution in a given space-time. As we noted, for some given space-times [17] the energy distribution according with the energy-momentum complex of Møller is the same as those calculated in the Einstein prescription. Our results sustain the conclusion of Lessner [27] that the Møller energy-momentum complex is an important concept of energy and momentum in general relativity. Also, the Møller energy-momentum complex allows to make the calculations in any coordinate system.

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